
Sky Lesson - Measuring the Moon's Size and Distance During a Total Lunar Eclipse with Earth's Shadow and a Tennis Ball

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The following exercise is from the original The Classroom Astronomer
Magazine December 2010 issue, updated for the May 2022 lunar eclipse.
We will be using this to find the Moon's distance during the lunar
eclipse!
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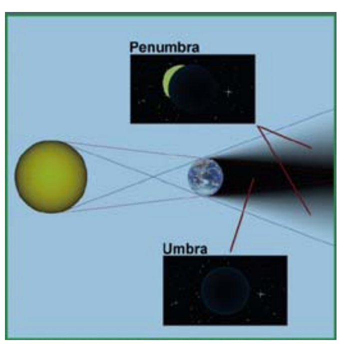
This technique can be done anyplace where the Moon can be watched through the beginning partial, total, and end partial phases of the eclipse. It can be recorded by drawing or photography.

The shadow of any spherical object in the solar system consists of a dark central cone called the Umbra and an outer shadow zone surrounding the cone called the Penumbra, where some sunlight shines directly into that space. The umbral shadow edges may appear sharp to the unaided eye but it isn't as sharp as it seems. This will cause some uncertainty in the measures, an important point all budding scientists should learn--no measurement is ever infinitely precise. We are attempting to measure the diameter of the umbral shadow at the distance of the Moon when it gets eclipsed this time. The Moon's distance varies because of its elliptical orbit. The Earth's shadow length also varies; the cone can be longer or shorter depending on whether we are close to the Sun or farther, though not as extremely as the Moon's.

The farther away the Moon is, the smaller the cross section of the umbra it cuts through. So, measuring the ratio of the size of the Moon in our sky to the size of the umbral shadow cross section in degrees during totality gives us a way to find the distance to the Moon at that time.



Courtesy Wikipedia Commons

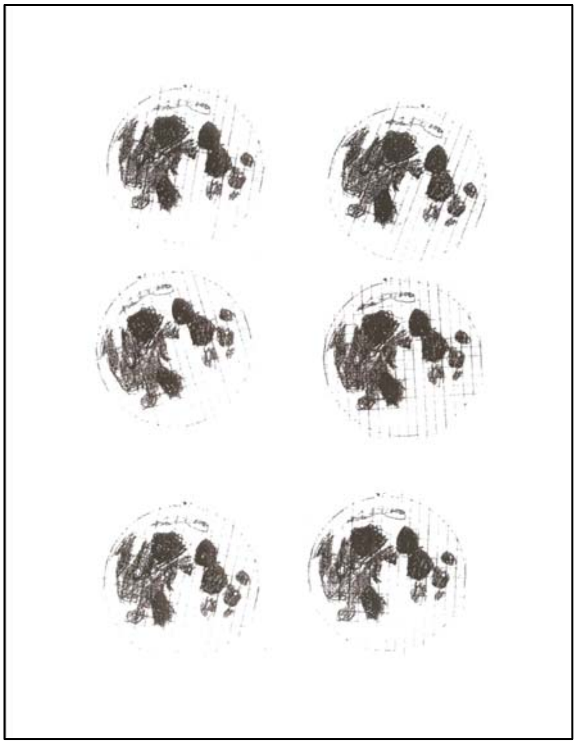
Here's what you will do to measure the Moon using the Shadow technique:

1. Take drawings or photographs of the Moon as it moves through the Earth's umbra (central) shadow.
2. After the eclipse ends, put your photographs or drawings on a timeline along the edge of a piece of graph paper (preferably).
3. Here's the challenging part: Using a drawing compass and adjusting its size AND where the 'center point' should be, find the circle that BEST fits the circular edge of the shadow as the Moon moved through the umbra.
4. Measure the diameter of your estimated shadow cross section in millimeters, and the diameter of a Moon image, dividing the first by the second to get a ratio value somewhere between 2.5 and 3.0. You will use this to find the Moon's distance and size.

Before The Eclipse

1. You MUST be within the zones where you can see all the total phases and at least some of the partial phases in order to do the Shadow Method.

2. Download the Moon drawing sheet and copy it to paper.



This is just an excerpt of the full sheet. See the upside-down three-toed footprint on the right (west side)? When you line up your drawings, that footprint should ALWAYS be to the right!

3. Print also the timeline sheet attached to the drawing sheet (see below for its image) and use or adapt this to another piece of paper. **The Moon moves its own diameter every hour so each hour mark MUST BE wide enough to just appear on the left and right edges of any Moon drawing image.**

   Extra Credit but Worth Doing:
   An interesting experiment to try near Sunset or Sunrise on a day before the eclipse happens is to **measure the ratio of the diameter of a spherical object (a tennis ball, a marble, a globe) and the length of its umbral shadow cone.** The umbra of the ball is easy to see on a screen when the screen is a meter or yard away from the ball but as the screen gets further away, the umbra fades into the growing penumbra (this is why we do this late or early in the day, we need the horizontal distance a shadow will go when the sun is low in the sky!) *One should find that, when you average several measures, the ratio of shadow-cone-length to sphere-diameter is pretty much the same no matter what kind of sphere you use!* The ratio is just a bit over 100. In fact, the actual equation should be:

Equation 1: Shadow Cone Length L = 108 x Diameter of sphere.
You can then scale this up to the size of Earth and calculate its shadow size to be
L(Earth) = 856,000 miles from Earth.

During The Eclipse

You must begin drawing the edge and coverage of the shadow on the Moon circles just as the partial phases begin, and end just after the partial phases end. **We suggest you make your observations every 30 minutes; overlaps are okay. Your drawings should draw the shadow edge as it appears over the lunar seas that you see with the naked eye (or low-power binoculars) and with the shadowed part shaded in.** It is very easy and tempting to draw the eclipse as it appears in the sky relative to the *horizon* but this would be wrong!!! Observers should rotate their sheet of paper with Moon images to *match the Moon* as it appears, particularly as to which maria (seas) are "up". Record the times of observations on a white part of the Moon circle. We suggest you NOT cut out the circles until after the eclipse.

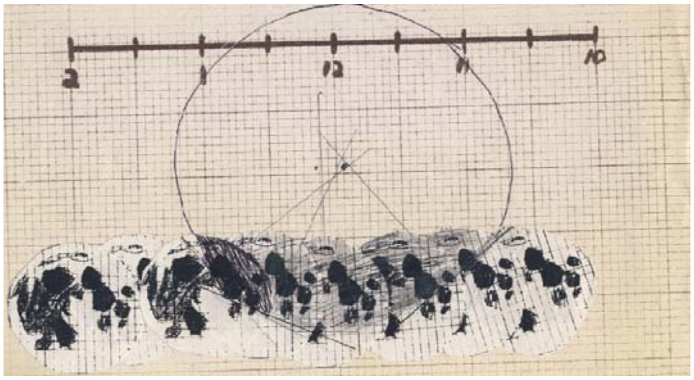
(You can also take individual photos during the eclipse and print them and cut them out and put them on a timeline. There are some photographers who can take a series of photographs on the same image (sheet of film or stored CCD image) and show the circular outline of the umbral shadow.

After The Eclipse

1. Cut out and put the Moon drawings on the paper with the time line. Mark your hours and half-hours on the timeline as appropriate for your time zone. If you drew the Moon every thirty minutes you should have Moon images overlapping. This is a good thing; you will be able to "average out" some of your drawing discrepancies.

   Note that though the Moon's features may appear to rotate as the Moon moves across the sky, they actually do not and all your lunar seas should always line up exactly the same way on the timeline - the seas that seem to form a three-toed footprint should *always* be on the right (west) side of each drawing.
2. You should see that the edge of the Earth's shadow has a circular form, but you will only see part of the circle. Your next job is to make the circle that best fits the visible shadow edge arcs. This may require several attempts to get it done well; you are doing a BEST FIT and it won't be exact or going through all parts of the arc perfectly. We find that if you make tangents around the drawn large circular edge at several places, and

then use a T-square to get perpendiculars to the tangent lines, you will find they roughly converge near a point. Adjusting your drawing compass in size and center location place, you will eventually find your 'best fit' circle.



Here are the Moon circles placed on a timeline paper. Notice that all of them have the "footprint" facing to the right (west). Also, see the tangent lines along the edge of the shadow and the perpendiculars from them that helped locate the center of the umbral cone cross-section....

3. Measure the Moon image diameter and the best-fit shadow circle diameter in millimeters. Divide the second value by the first; you should get a ratio between 2.5 and 3.0 (usually around 2.7). Calculate the shadow circle diameter in degrees by knowing the Moon is ~0.5 degrees and multiplying that by the ratio you just determined (e.g. 0.5 x 2.7 = 1.35).

Activity 1 — The Moon's Distance.

Any object has an angular size in radians equal to its diameter in miles divided by its distance from us in miles. Now, as the distance gets larger, the shadow cone cross section (XSD) will get smaller. It starts at 7926 miles at the bottom of the cone (at Earth), goes to zero at the end, and thus has an proportionally intermediate size between the extremes. For example, fly 25% of the 856,000 miles away and you'll find Earth's shadow's cross section is 100-25%, or 75%, the size of Earth, down to 5945 miles across. This we can call

Equation 2: XSD = 7926 x (1-f), where f is how far into the shadow cone you are from Earth

—above it was 25%. Conversely if you can measure the size of the cross section in miles, you can determine how far away you are! That's where the Moon and its eclipse comes in. The ancient Greeks knew to use them to find the Moon's distance.

We've measured the size of the umbra, (typically it is 1.33 degrees, close to the value in our example above). In the box below we take that basic definition of angular size of an object and substitute Equation 2 on top and Equation 1 on the bottom, and generate Equation 3, the f equation at the end of the chain of mathematics. That is what we need.

$$\text{Angular Size (A)} = \frac{\text{XSD (cross sectional diameter)}}{\text{Distance to Cross Section}} = \frac{7926 \times (1-f)}{7926 \times 109 \times f} \Rightarrow f = \frac{1}{[(A * 108)+1]} \quad \text{(Eq. 3)}$$

Let's take a **typical** scenario. The shadow cone cross section size we found to be 1.35 degrees. (If you did the tennis-ball extra credit assignment, use that value you found, otherwise, use 108, and your umbra size from your observations and the circle you made!) To convert any degree value to radians, divide it by 57.3 degrees per radian; this makes the angular size of the shadow .0233 radians. Plugging this value for A into Equation 3 gets us *f* = 1/ [(0.0233 x 108]+1) or 0.282, 28% of the way from Earth to the end of the cone. Multiply *f* by the shadow length 856,000 miles makes the Moon about 241,000 miles away. We have found the Moon's distance!

Use THIS eclipse's observed values to get your data. Above is just a routine example.

Activity 2 — The Moon's Diameter

This is simpler. Knowing that the Moon (and the umbral cross section) is, say, 28% of the way out, the size of the shadow we see on the Moon is (from Equation 2): 7926 * 1-*f*, or 7926 x (1-.28), yielding **5706** miles across.
As the shadow is 2.67 times the Moon's size, the Moon must be 5706 / 2.67, or 2137 miles across, about 1% smaller than its actual 2160-mile size! We clearly can determine size and distance of the Moon to within a few percent accuracy! Use your eclipse values to get the March 2022 values.

The best science technique is to get a group to do this, and do statistics, i.e. get a mean value and graph the spread, and see how close to known values you get, and remember, the Moon's orbit IS elliptical so it won't be the average Earth-Moon distance. But the Moon's diameter doesn't change <g>.

How clever those ancient Greek astronomers were!

Sky Lesson - Results of Measuring the Moon's Size and Distance During a Total Lunar Eclipse

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In the last issue we wrote about how to use a tennis ball and drawings
of the Earth's shadow on the Moon to determine the Moon's distance and
size.  In THIS issue we use the results of our observations of the May
15-16, 2020 total lunar eclipse to show real results.  You can then
refer to this again during the upcoming partial lunar eclipse, which
might be even better than a total for this exercise, in November!!
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The May 2022 total lunar eclipse was very nearly central so a) it was really dark as well as red—called a Blood Moon, it was more an Old Blood Clot Moon, and b) the arcs of the Earth's shadow were only upon entrance and exit, making the determination of its size a bit more difficult than an eclipse in which the Moon skirts the umbra all the way. Furthermore, the Moon was near perigee, so the umbra was quite large, larger than average, and so was the Moon. Still, the exercise is doable.

The farther away the Moon is, the smaller the umbral cross section it cuts through. So, measuring the ratio of the Moon size in our sky to the umbral shadow cross section size in degrees during totality gives us a way to find the Moon's distance at that time.

In review, here's what you had to do to measure the Moon using this Shadow technique:

1. Take drawings or photographs of the Moon as it moves through the Earth's umbra (central) shadow.

2. After the eclipse ends, put your photographs or drawings on a timeline along the edge of a piece of graph paper (preferably).
3. Here's the challenging part: Using a drawing compass and adjusting its size AND where the 'center point' should be, find the circle that BEST fits the circular edge of the shadow as the Moon moved through the umbra.
4. Measure the diameter of your estimated shadow cross section in millimeters, and the diameter of a Moon image, dividing the first by the second to get a ratio value usually somewhere between 2.5 and 3.0. You will use this to find the Moon's distance and size.

Unfortunately for yours truly, HIS location was one of the few in Alabama that was under the influence of thunderstorms! The only parts of the actual eclipse that were visible were those during the end partial phases, after totality had ended. Insufficient to do the experiment observationally. However, the event was live-streamed by multiple sites and we chose to use www.timeanddate.com. It gave views from multiple telescope sites, all imaged at the same size and with a time stamp visible. We took screenshots as close to every hour and half-hour as possible, provided the image showed a clear shadow edge, and preferably coloration.

You may need to refer to the previous issue for derivations of the mathematics and procedures.

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Equation 1: Shadow Cone Length L = 108 x Diameter of sphere.  L(Earth)
= 856,000 miles from Earth.
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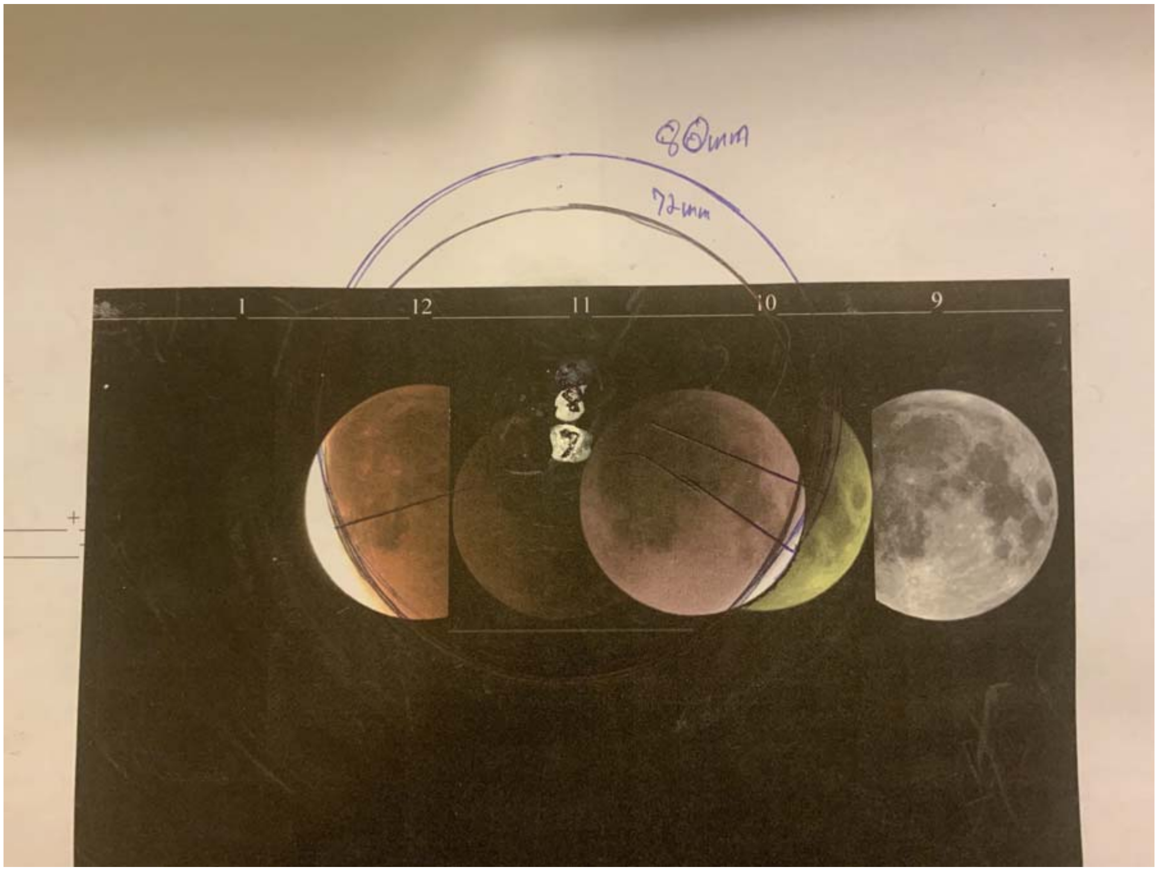
After The Eclipse

1. Instead of using our timeline and cutting out our hand-drawn Moon shadow drawings and pasting them on it, we instead took our live-streamed screenshots, cropped off all but the disk and put them onto a desktop publishing page (we are old-fashioned, we used Microsoft Publisher). Our timeline page was a line with one-inch-apart vertical hour marks in local Alabama time. We sized our Moon images to fit exactly within the hour marks since, to a close approximation, the Moon should move its own width in one hour. The usable intermediate photos were over- or underlain between the hour photos, clearest images on top.

   Note that though the Moon's features may appear to rotate as the Moon moves across the sky, they actually do not and all your lunar seas should always line up exactly the same way on the timeline - the seas that seem to form a three-toed footprint should *always* be on the right (west) side of each drawing. We rotated images and used a horizontal line to make sure all the seas were horizontally lined up.

2. Clearly, we see that the edge of the Earth's shadow has a circular form, but we only see two parts of the circle. Our next job is to make the circle that best fits the visible shadow edge arcs. This required several attempts to get it done well; it IS a BEST FIT after all and it won't be exact or going through all parts of the arc perfectly. In the past we usually found that if you make tangents around the drawn large circular edge at several places, and then use a T-square to get perpendiculars to the tangent lines, you will find they roughly converge near a point. Adjusting your drawing compass in size and center location place, one will eventually find your 'best fit' circle. We initially found a smaller, 72mm circle, but then found an 80mm circle seemed to be a better fit.



3. The measured Moon image diameter on paper was 34mm. The ratio of umbra and Moon diameters, usually between 2.5 and 3.0, was 2.35. Then the calculated shadow circle diameter in degrees, determined by knowing the Moon being near perigee was not its usual ~0.5-degrees but actually 0.55-degrees, was found by multiplying that by the ratio just determined, i.e 0.55 x 2.35 = 1.29 degrees).

Activity 1 — The Moon's Distance.

Any object has an angular size in radians equal to its diameter in miles divided by its distance from us in miles. Now, as the distance gets larger, the shadow cone cross section (XSD) will get smaller. It starts at 7926 miles at the bottom of the cone (at Earth), goes to zero at the end, and thus has an proportionally intermediate size between the extremes. At 25% of the 856,000 miles away, you'll find Earth's shadow's cross section is 100-25%, or 75%, the size of Earth, or 5945 miles across. Thus:

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Equation 2: XSD = 7926 x (1-f), where f is how far into the shadow cone
you are from Earth — above it was 25%.
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Conversely if you can measure the size of the cross section in miles, you can determine how far away it is! That's where the Moon and its eclipse comes in.

We've measured the angular size of the umbra. In the box below we take that basic definition of angular size of an object and substitute Equation 2 on top and Equation 1 on the bottom, and generate Equation 3, the *f* equation at the end of the chain of mathematics. That is what we need.

$$\text{Angular Size (A)} = \frac{\text{XSD (cross sectional diameter)}}{\text{Distance to Cross Section}} = \frac{7926 \times (1-f)}{7926 \times 109 \times f} \Rightarrow f = \frac{1}{[(A * 108)+1]} \quad \text{(Eq. 3)}$$

Let’s put in our observed values.

- The shadow cone cross section size we found to be 1.29 degrees. To convert any degree value to radians, divide it by 57.3 degrees per radian; this makes the angular size of the shadow .02256 radians.
- Plugging this value for A into Equation 3 gets us *f* = 1/ [(0.02256 x 108]+1), getting 29% of the way from Earth to the end of the cone.
- Multiply *f* by the shadow length 856,000 miles makes the Moon about 248,240 miles away. We have found the Moon’s distance!

In reality, the Moon near perigee was at 225,757 miles distant. We are too far by about 10%. Not too bad considering the uncertainties, which were the printed Moon sizes and placements on the timeline, the inch marker placements, the umbral fuzziness, and the center point placement.

Activity 2 — The Moon’s Diameter

This is simpler. Knowing that the Moon (and the umbral cross section) is 29% of the way out, the size of the shadow we see on the Moon is (from Equation 2): 7926 * 1-*f*, or 7926 x (1-.29), yielding **5627** miles across. As the shadow is 2.35 times the Moon’s size, the Moon must be 5627 / 2.35, or 2394 miles across, about 11% larger than its actual 2160-mile size!

We clearly can determine size and distance of the Moon to within a few percent accuracy! How clever those ancient Greek astronomers were!

How close do you get with your (or your group’s) observations, by drawings or by photographs?

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Articles written by Dr. Larry Krumenaker