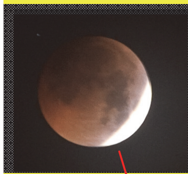


The Lunar Parallax and December's International Measure The Moon Night

- Larry Krumenaker

The ideal collaborative inquiry activity, joining forces to measure the Moon's distance via parallax. Join in for the December eclipse!

So simple in concept even the ancient Greek astronomers knew *this*. For them, though, it was an abstract mind experiment. It couldn't be done in the real world. Their real world. But we can do *this*, today, in our reality.

This...is measure the Moon's distance and size. All it requires are two observations. The difficulty the ancients could not overcome was that the two observations had to be taken, and on opposite sides of the Earth. The procedure is called taking a parallax measurement and surveyors do it all the time. Parallax is simple to show: point your index finger upwards and at a distant object (a light post, something on the wall) and while doing that, put your hand straight out as far forward as you can. Look at your finger with just one eye. Now, switch eyes. The finger should appear to shift against the background by some angle. Switch back to your other eye. Move your pointing finger halfway in towards your eye, still pointing at the object. Switch eyes again. The shifting angle should be bigger. The bigger the shift, the closer the object.

What if you could somehow take your two eyes and move them farther apart? (Kids, don't try this at home….). Then the angle would get larger and you could also measure things that are farther away. You are limited to how small an angle —the parallax—your 3-inch-separated eyes can detect.

The Greeks knew this. Because they detected no parallax for the stars, they knew stars—and everything else—were beyond Earth's atmosphere at least. Even the Moon, presumably the closest object to us, had no detectable parallax from the Grecian city-states. What was needed was a larger baseline, the line between the two viewing places. The largest one we have to use for the Moon is the Earth's diameter. Unfortunately, Chinese food—and astronomers—were unknown to the ancient Greeks and even if they were known, no communication was possible to set up this experiment. Today, with email, the Web, and other means of communication, we can do *this*. And last June, it was tried.

Measuring the June Lunar Eclipse

TCA contacted a number of persons last May to try this experiment from opposite sides of the upcoming June 2011 total lunar eclipse viewing zone. The borders of visibility were central Europe and the eastern edges of Asia and Australia. These would provide the largest possible baseline to have the Moon visible from two places at the same time. The key requirements asked of the photographers were: take every photo at the exact same moment—every minute ending with a 5 (:05, :15, 0:25, etc.) during a common period of visibility overlap (the Moon rises during the European photo session while setting during the Asian views), make sure there are stars visible in the photo—preferably a minimum of around 3 degrees or more of sky around the Moon, and the photo shows lunar maria features so we can match up Moon images.

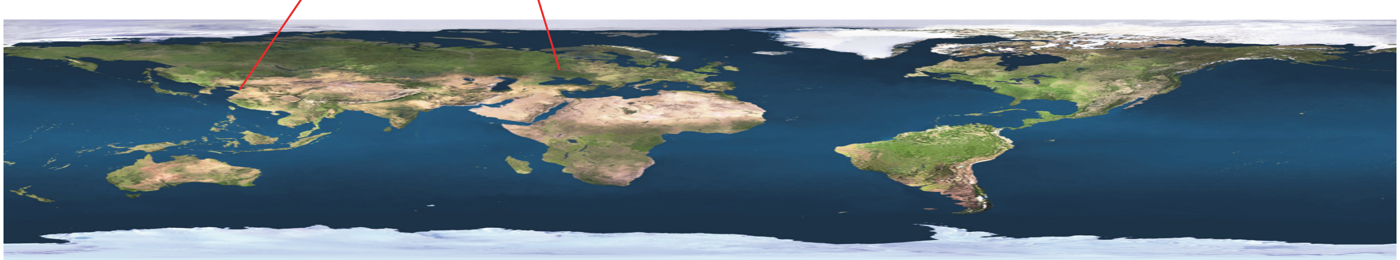


Figure 1. Worm's eye view of the Earth's surface with lines of sight from two eclipse observers' positions to their photographs of last June's total lunar eclipse. Not to scale, the arc is the lunar parallax angle, really only about 1^{o}, not the large angle shown.

At first it seemed Nature was against them. The Italians got beautiful clear skies but the Aussies and Koreans were clouded out. A hunt ensued for other photographers with better luck, and we found two in Turkey and in China who kindly let us use their photos. Thank goodness today we *can* communicate with the other sides of the world!

As can be seen on this issue's cover photo, which combines the Italy- and China-based photographs (the latter was taken in China by a Korean amateur photographer working there but we'll call it the Chinese photo from now on), there is quite a noticeable shift! The Moon itself is generally one-half degree [30' (minutes of arc)] in size, though it can vary by 1-2' either way which is a not-noticeable-to-the-eye amount. The lunar parallax is easily twice the Moon's angular size, about one degree! But how far away the Moon is in miles with that kind of shift requires that we know how far apart the photographers are. That's not as obvious as it seems.

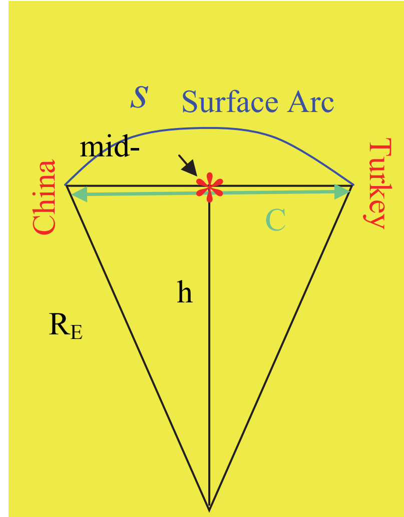


Figure 2. The geometry of observing from two places on Earth's surface. As the article will show, the surface distance between Qingdao, China and Adana, Turkey is 4549 miles, the real distance, or chord, is 4303 miles, and the distance of the midpoint to the Earth's center is 3308 miles. Radius of the Earth is 3963 miles.

One can, theoretically, put a couple of pins in a globe at the two locations, use a measuring tape and the globe's scale and find the miles or kilometers apart they are but this *isn't* how far apart they really are! This is how far along the *surface* of the Earth they are! We really need the straight-line, through-the-Earth distance to get the true measure.

It turns out there are two things we need to find that true mileage-apart measure. First, we really do need the surface distance measure. If one doesn't know the latitude and longitude of the two places and/or doesn't want to do spherical trigonometry to find the arc length between the points, there is at least one website that can do it for you. GeoBytes City Distance Tool has the names of thousands of locations on the Earth and can calculate the distance between them; it is found at http://www.geobytes.com/CityDistanceTool.htm .

Then, second, the straight-line distance can be calculated with a simple formula. A look into any decent geometry reference, or in our case, the Standard Mathematical Tables book from the CRC (Chemical Rubber Company) scientific reference, found the graphic reproduced above and the equations to find the straight-line distance between the two points (called the Chord) that are on a surface arc.

(1) C (the Chord) = $2R_E \times \sin(s/2R_E)$,
where R_E is the Earth's Radius and s is the surface arc between the places.

It turns out that observing from the extreme ends of the eclipse visibility zone are not good places because both extremes have too much twilight. We can't actually use a full-Earth-diameter baseline, 12,756 kilometers or 7926 miles. Because the places have to be as far apart as possible and yet dark enough to show stars will be closer together than this, our baseline will also be less. While the Italian photos showed some stars, we found better matches with photographs from China and Turkey.

First, The Baseline

An Kwang Jin's photo (Figure 3a) was taken in Qingdao, China, where he works. Seçil Berna Kuzu is a doctoral student in molecular biology and an avid amateur photographer. Her picture (Figure 3b) was taken in Adana, Turkey. Both places are listed in City Distance Tool website. The distance apart on the surface of the Earth for them is 4549 miles. Using Equation (1) above, and substituting 4549 for s and using the Earth's diameter, divided in half, for the radius of Earth R_E, the distance C is 4303 miles, a little over half the Earth's diameter.

Figure 3a. The nearly-eclipsed June Full Moon, and the star 51 Ophiuchi, from China (photo courtesy An Kwang Jin) ...

Notice that the Moons are rotated from each other with reference to the horizons. As any astute observer knows, the Moon and constellations and any extended objects are "tilted" counterclockwise from their "up and down" orientations as they rise in mid-northern latitudes and rotate in orientation through that straight up position until they become "tilted" clockwise as it sets. So we shall have to correct for the rotations. Note the bright star to the upper left of the Moon in the Chinese photo. That circled star is 51 Ophiuchi. You'll find it also in the Turkish photo, only to the right of the Moon! Somewhere in between, in the Middle East, 51 Oph was behind or just next to the Moon.

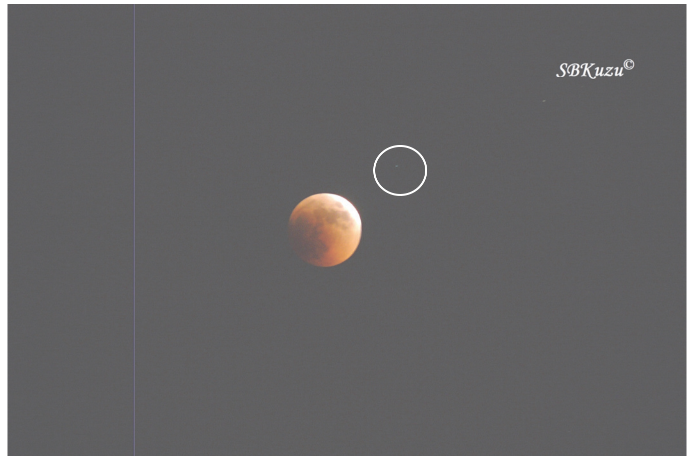


Figure 3b. ...and from Turkey, courtesy Seçil Kuzu.

Second, The Parallax Angle

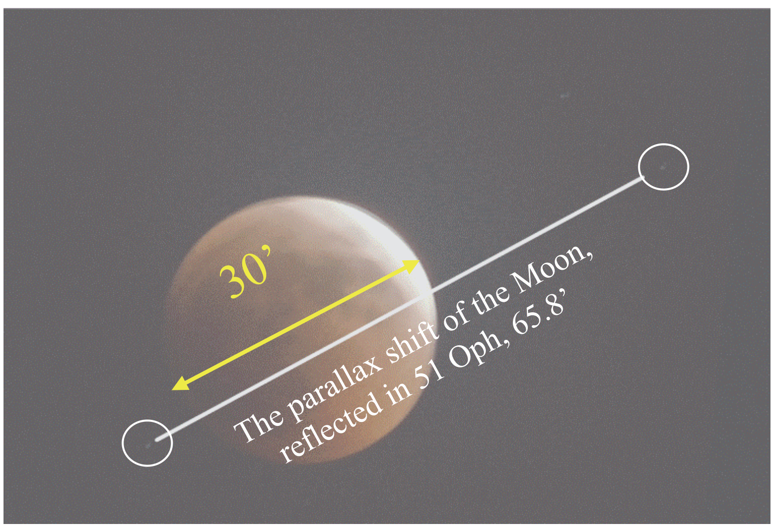


Figure 4. Figures 3a and 3b rotated, overlain, and a line connecting the same star seen in both photos shows the Moon shifted just over one degree as seen from China and Turkey. (Contrast has been adjusted here and in 3b!).

There are two ways to measure the lunar parallax. One way is to overlay the two photos so that all the stars in each are on top of each other. This would show the Moon in two places over the single identifiable star-field, like our cover photo. Easier and more doable is a second way, which we did, which was to overlay the two photos' Moon images, stretching one for size and rotating as needed until they were indistinguishable (Figure 4). Knowing that the Moon has a 30'size means getting the photo scale is simple. Measure the overlain Moon image diameter with a millimeter scale, divide the number of millimeters into the 30' of arc, and there's the scale. Now to get the actual parallax, find the two images of 51 Ophiuchi in the overlapped image, one to the left of the Moon, one to the right. Measure their distance on your photo, apply your scale, and voila!, you have the minutes of arc of the lunar parallax!

Last, What is the Moon's Distance, and Size?

The angle as seen between the Turkish and Chinese photos was 65.8' (minutes of arc) or 1.097 degrees. This is the parallax but...the triangle made is a bit complicated because it hasn't a right angle in it. That angle would be found at the midpoint *in* the Earth halfway between the two places, cutting the angle shift in half. So if we have our triangle's point being 1.097 degrees divided by 2, or 0.5485 degrees, and the baseline (the short side of the triangle) now half of 4303 miles, or 2151.5 miles, the basic definition of the trigonometry tangent function finds the distance to the Moon. The equation (which includes converting the degrees to radians and using the known approximation that the tangent of a small angle in radians is about equal to the small angle itself) is:

(2) Distance midpoint to Moon = (1/2 Chord) / (parallax/2/57.3 degrees per radian)

Filling in the numbers, the Moon's distance from the midpoint is [2215 miles/(1.097 / 2 / 57.3)] or 224,753 miles away. Even without accounting for the fact that our baseline is inside the Earth a bit, we are only 3% off of the moon's real distance of 231,483 miles! If we do add in the distance h, the distance from the midpoint to the Earth's center as shown in Figure 2, then we use Equation 3,

(3) Distance midpoint to Earth's center $h = 0.5 \times \text{square root}(4 \times R_E^2 - C^2)$

to add in some 3328 more miles which gives us a total of **229,081** miles, a mere 1.5% difference.

The Moon is 30' across or 45% the amount of the parallax, equivalent to 45% of our baseline. Forty-five percent our baseline's 4303 miles is **1937** miles. The real size of the Moon is 2140 miles, we're off by about 9.5%.

Things to Watch Out For Next Time

To avoid large errors in the calculation of the Moon's distance, first, the midpoint should be in line with the Moon at that moment of photography. Longitudinally, June's would have been under India; happily, Turkey and China are about equally far from India. Otherwise, the line from Earth's center to midpoint to Moon is not straight! Furthermore, the two observers have to be either at about the same latitude (these two are), or one must be the same amount South as the other is North. If either condition fails, the calculated distance will be too short. It is key that the photographs be wide enough to capture the SAME stars yet have large enough scale to show lunar details. They also have to be taken within a minute of each other so any umbral edge will be crossing the same lunar seas or craters! If not, the measured shift will be thoroughly wrong.

Lastly, the observers can not be TOO far apart, else they won't both get photographs with stars, nor TOO close, because the percentage error starts to increase when the two observers are within 45 degrees of longitude of each other. Star shifting will be only as far apart as the Moon's size, and possibly lost in lunar glare.

One *can* simplistically use the surface arc distance and still get reasonable, though of course a bit less accurate, results. For those not-so-mathematically inclined, this still makes it a doable project. TCA

Taking Photos During Totality

There are four key factors for taking photographs of a totally (or nearly totally) eclipsed Moon: Lens focal length, "film" speed or ISO, exposure time and f/ratio. There is some dependence on the resolution of the image, whether you are taking high resolution (bigger files, too) or low-res. Our three photos varied from resolutions of around 3000K by 2000 pixels down to 900 by 600 pixels.

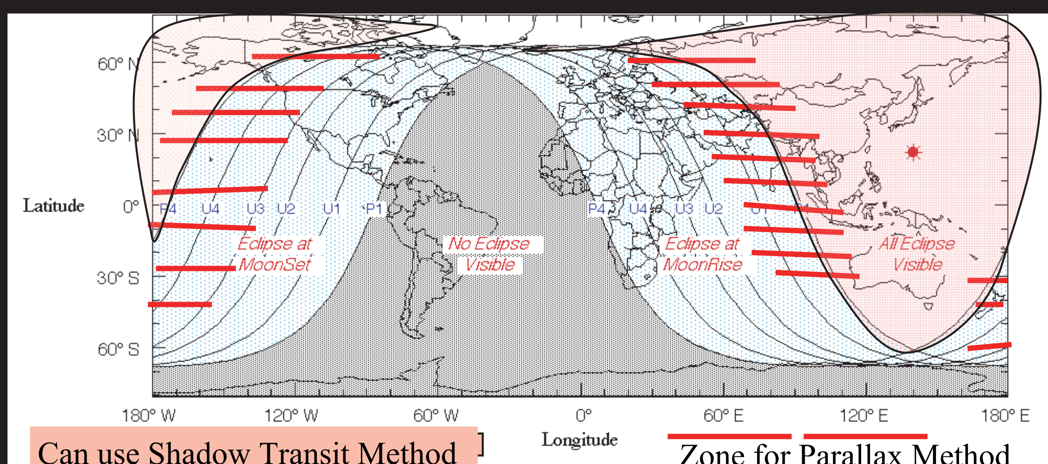
Kuzu used an ISO setting of 200, F/3.3 with a 67mm lens (many cameras commonly come with a 50mm normal lens, a wide angle is less, 20mm, and macro/telephone lenses go up from 125mm). Her exposure was 1 second.

An's photo was taken with an f/5, ISO 400, and a 190 mm telephoto, more a close-up but at a low resolution setting, also around one second (longer means the Moon will trail (smear) on the exposure).

The Italian panorama was a ten-second exposure, ISO 1250, the 70mm lens set at f/6.3.

More detailed information from the three photographers can be found online at www.MeasureTheMoon.Org.

Measure the Moon's distance and size with December's lunar eclipse!
Collaborate with students and observers in multiple countries.



The last opportunity before 2014!

Base image courtesy Jet Propulsion Laboratory

SUMMARY

1. Take photographs of the eclipsed Moon, showing stars, from two widely separated locations (1000s of miles apart) at the same moment.
2. Overlay the photos, matching up lunar surface features.
3. Measure the shift of one star between the two places using the Moon's 30' diameter as your ruler.
4. Use the Geobytes website to find out how many miles or kilometers apart the two observers were.
5. Use the three equations to find out the real distance between the places through the Earth (the Chord), the distance to the Earth's center, and the Moon's distance.