Transit of Venus starting photo taken in Marietta, GA June 5, 2012
Transit of Venus ending photo taken in Marietta, GA June 5, 2012
Transit of Venus starting photo taken in Cairns, Australia June 6, 2012 (local date)
The Classroom Astronomer Magazine’s Transit of Venus Lab Exercise

Transit of Venus ending photo taken in Cairns, Australia June 6, 2012 (local date)
To Determine the Distance to Venus, and the Value of the Astronomical Unit (AU)

While determining the distance to Venus is in principle a mere geometrical exercise, a surveyor’s triangular measurement, the fact is that it is a bit difficult because of the tiny angle Venus makes as seen from two different places on the Earth. This, the parallax angle, is the tip of a very narrow triangle and the base of the triangle is the north-south distance between the two observation sites—THROUGH the earth, not over the surface. This requires a bit more math than the usual geometry problem. Finally, while normally we might be able to ignore an additional factor, the sun’s parallax angle from those two sites, the tininess of Venus’ angle makes the Sun’s own parallax amount impossible to ignore. To do so can cause the error to be quite large.

The procedure to be followed, originally created by Edmond Halley back in the 1700’s, is based on Sten Odenwald’s Space Math for the Transit of Venus, found on http://spacemath.gsfc.nasa.gov. Here we will excerpt the necessary instructions without proof or derivation, and give you the results we determined. Your results should be similar in a classroom exercise but, as you will see, there are factors that can cause great amounts of error.

The overall procedure is:

1) Overlay the pairs of photos from the same places and draw a line that marks the path of Venus across the Sun’s face at that point, called the chord.

2) Overlay the overlain pairs so that you match up the sunspots on the photos on top of each other. This will show the two chords are not on top of each other. This is the observed chord separation, the observational parallax of the transit and nominally that tip of the triangle, but the solar parallax and other factors are not yet taken into account.

3) Measure the distance between the chords on the combined four photo overlay. Also measure the diameter of the Sun, edge to edge. Determine the angular distance between the two chords as a ratio of the linear distances between the chords and the diameter of the Sun.

4) Determine the baseline distance between the two latitudes, pretending they are on the same longitude.

5) Input the parameters into the equations to be given, to determine the actual parallax of Venus, the linear distance to Venus knowing also that on this date it was 0.28 AU from Earth…

6) …and then prorate this to one full AU, thus determining the scale of the solar system.

The following facts are known:
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The Georgia observations were done in Marietta, GA, at a latitude of +34.02 degrees. Only about an hour of the transit was visible, due to clouds, local obstructions and then Sunset. The photographs were not taken with the exact same setup throughout the experience, hence the Sun is slightly oval in one photo, etc. Venus is visible in the first photo on the Sun’s edge, at about the 1 o’clock position.

The Australian pair was done in Cairns, at a latitude of -16.95 degrees. The full transit was visible here, from start to end, and was observed with the same equipment throughout.

The Sun’s angular diameter in the sky that day was 32.6 minutes of arc – there are 60 minutes of arc in a degree and 60 seconds of arc per minute of arc or 1956 seconds of arc.

Earth is 12,756 kilometers in diameter. Venus is 0.28 Astronomical Units (AU) from the Earth.

Advice: It would be best to capture the transit images into some graphics software, like Photoshop, where photos can be placed on top of each other and the top one is transparent so you can adjust the size and orientations of the photos to match up the various sunspot images. An alternative is to print these on clear plastic, like those for old-tech overhead projectors. Have a good ruler with a millimeter scale. The bigger the photo images, the more accurate your measures will be. Finally, if your chords do not end up parallel, start over.

For those who do not have the means to make the overlays, we do include our own overlay photo, at the end of the exercise.

Measuring the baseline

The distance between two sites on the same longitude, in kilometers and through the Earth is:

\[ L = 12756 \sin \left( \frac{\theta b - \theta a}{2} \right) \]

where the two latitudes are used for the thetas, north – south and south is negative. We don’t worry about the difference in longitudes because we care about the north-south difference only when we are doing parallel tracks. We would only be concerned if we were doing a parallax comparison at a single moment in time.

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Photos courtesy of Larry Krumenaker (Georgia) and the Gloria Project (Australia)
Measuring the chord separation

First you have to measure the size of the diameter of the Sun on your combined photos with two chords. Use the largest possible image you can make and measure in millimeters. Then measure the distance in millimeters between the chords. It is best to make measures at several places along the whole track and average them. Solve using proration

Given the sun’s size in seconds of arc, use proportions to find the distance in seconds of arc between the two chords.

Measuring the solar parallax

Next, determine the solar parallax, i.e. how far the Sun itself shifted in the sky because of observing it from two places. However, the Sun is too brilliant to see its shifting position against background stars. In effect, today, we know the solar parallax because we measured the real distances to Venus, Mars and some asteroids and can work the problem backwards to determine what we could have measured if only we could see it.

The largest possible parallax, from extreme opposite edges of the Earth, separated by 12756 km, would be 17.6 arc-seconds. So, just as before, we prorate the baseline with the Earth’s diameter to get what we should see. take the baseline between Cairns and Marietta, divide by the Earth’s diameter, and multiply by 17.6 arc-seconds.

Determining Venus’ parallax

Pv (the parallax of Venus) is equal to the observed chord separation parallax plus twice the solar parallax (twice because it affects both observers’ observations). Add your determination for the chord separation in seconds of arc to two times the value of the solar parallax you determined above.

Determining Venus’ Actual Distance

The actual distance is determined by the following equation derived from basic triangle math: \( \text{Pv} = 206265 \left( \frac{\text{H}}{\text{D}} \right) \), where H is the baseline distance, Pv is the angle of Venus’ parallax and D is the distance to Venus that we want. Solve for D!

Determining the Astronomical Unit

Venus on this date is 0.28AU from Earth. A simple solving of the proportion will get the observed value for 1 AU:

\[
\text{D} = \frac{\text{AU}}{0.28} = \frac{\text{D}}{1} \\
\text{where D is the value you determined for Venus’ distance.}
\]
Our Results

1) The difference in latitude between Marietta and Cairns is 49.97 degrees. The baseline is 12756km X sin(49.97/2), or 5388 km.

2) On our original screen image (different undoubtedly from your image size!) we measured the Sun’s diameter as 119 mm and the chord separation as 2.5 mm.

The observed chord separation is:

\[
\text{Measured sun size} \quad \text{chord separation} \\
\frac{119\text{mm}}{1956\text{arc-second}} = \frac{2.5\text{mm}}{41.1\text{arc second}}
\]

3) The solar parallax is 17.6 x baseline/Earth’s Diameter, or 17.6 x (5388/12756), or 7.43 arc-seconds.

4) Venus’ parallax = observed chord separation size in arc-sec + 2 x solar parallax.
   \[= 41.1 + 2 \times 7.43 = 41.1 + 14.9 \text{ (rounded)} = 56.0 \text{ arc seconds} \]
   However, the geometry here is not that of a right triangle (it is nearly an isosceles triangle) so we need to use HALF this value for the calculation of the actual distance.

5) Venus distance is found from \(P_v = 206265 \times \frac{H}{D}\). So \(56.0 = 206265 \times \frac{5388}{D}\).
   Solving for \(D\), \(D = 206265 \times \frac{H}{P_v}\), or \(D = 206265 \times 5388 / 28.0\), yielding \(39,691,279\ km\).

6) The Astronomical Unit is 1/0.28 times larger than this so the \(\text{AU} = 141,754,568\ km\).
   This is 5% off the true value of nearly 149 million kilometers.

These values are different from what was printed in TCA 12’s article due to the ability after deadline to refine the values.
An image of all four Sun photos from Cairns and Marietta during the Transit of Venus with tracks drawn on the image.