Illuminating Science Through Darkness 3 - Caught! Shadow Reveals Moon's Size and Distance

- Larry Krumenaker

Like a ghost caught in a flashlight, the Moon catches earth's invisible shadow and reveals its size and the Moon's distance.

It's a rather scary sight, when you think about it. Normally the full Moon lights up the sky with great brilliance and, suddenly, a circular bloody or oily stain appears and envelopes the Moon, causing darkness where there should be brightness. There is a dark zone in the sky that is over a degree in diameter! . No wonder less enlightened civilizations in the past (and present, too) freaked out over lunar eclipses. Columbus used it to get survival groceries from primitive Indian tribes in the Caribbean. Turkish fort defenders fell to Arab raiders during a World War I battle because of panic from a Ramadan lunar eclipse. But for astronomy teachers it is a chance to put some perspective to our little part of the solar system, to make measurements about the Moon.

Recall that we can find the size of the Earth via Eratosthenes experiments (see the article on Page 16 of this issue) and that previously, in Part 1 of this trilogy of articles (Page 20), we found that the average length of any shadow cone at the Earth's distance is 108 times as big as the sphere casting the shadow. The shadow cone of the earth has to be 108 times longer than the earth's diameter. Let's call the general idea **Equation 1: Length L = 108 x Diameter of sphere.** This puts the point of the cone at around 856,000 miles away. The *cross sectional diameter* (XSD) of the cone starts at 7926 miles at the bottom of the cone (at earth), goes to zero at the end, and thus has an proportionally intermediate size between the extremes—it gets smaller the farther we go away from Earth. For example, fly 25% of the 856,000 miles away and you'll find earth's shadow's cross section is 100-25%, or 75%, the size of Earth, down to 5945 miles across. This we can call **Equation 2: XSD = 7926 x (1-*f*)** where *f* is how far into the shadow cone you are from Earth —above it was 25%. Conversely if you can measure the size of the cross section in miles, you can determine how far away you are! That's where the Moon and its eclipse comes in. The ancient Greeks knew to use them to find the Moon's distance.

Activity 1 — Finding and measuring the earth's shadow

Figure 1 is a photo of a sheet of Moon images (a full size sheet can be found on the *Classroom Astronomer* website). Put them on a clipboard and bring some pencils, gray or colored, and proceed to draw the Moon's shadow on the lunar features every thirty minutes, starting when the Moon is predicted to hit the penumbra or outer shadow of the Earth. You won't see it, it's too diffuse. So the Moon drawing will have nothing on it but a time. Somewhere between 30 minutes and one hour afterwards, one may see the penumbra and can draw it but soon the darker umbral shadow will show up. The dark lunar seas or maria make good reference points to draw the shadow's edge. Perhaps you may find some color at some times and you can use colored pencils to locate them on the Moon as well.

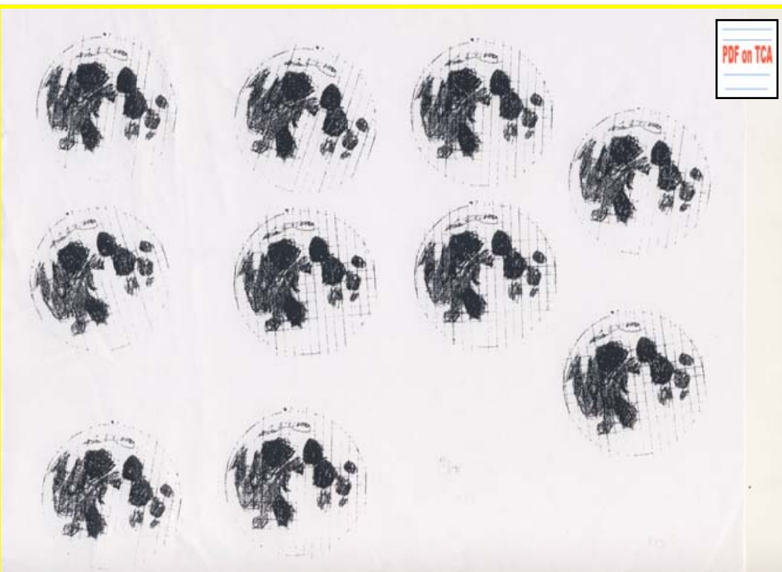
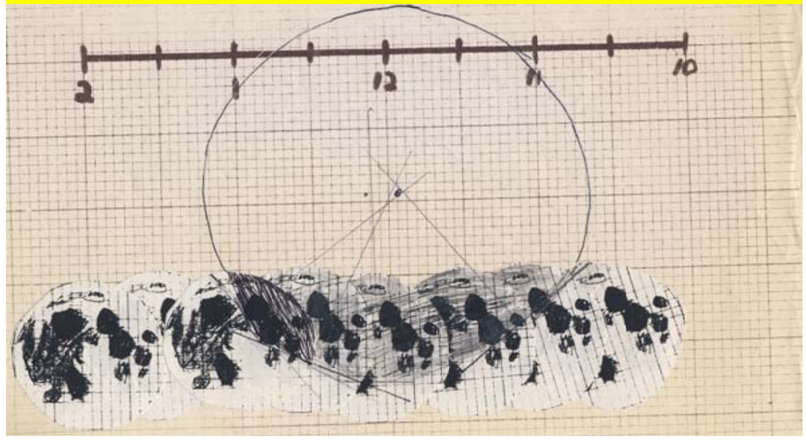


Figure 1. Identical Full Moon drawings that can be used to trace Earth's shadow's locations during lunar

After the eclipse ends, the drawings need to be cut out and placed on a piece of graph paper with a time line on it. To make the timeline, recall that the Moon moves its own apparent diameter every hour so you can make your timeline by putting two Moon images next to each other, not overlapping, and the left and right edges of the pair show the length of two hour son your graph paper. Make a time line long enough to cover the partial and total phases of the lunar eclipse.

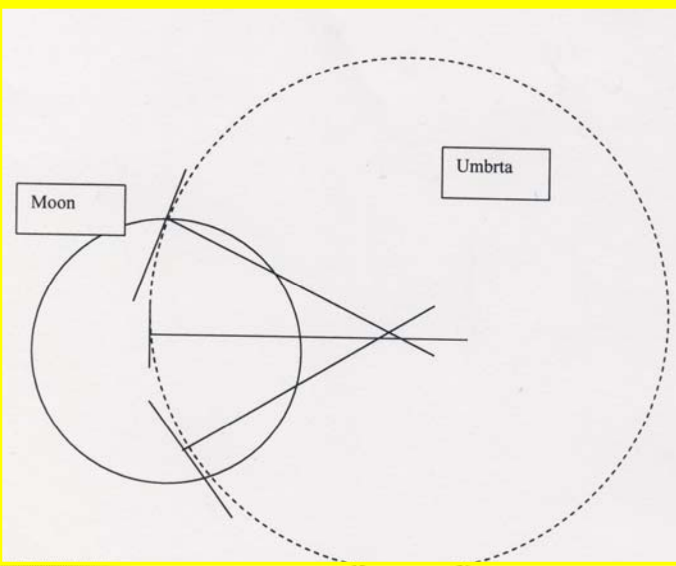
Figure 2. Moon drawings showing umbra shading from a partial lunar eclipse, properly placed on a timeline, and the umbral circle estimated.



Then, one by one and in order of time, put the Moon drawings in line with the time line but along the other edge of the paper so you can have room to draw the umbra's circle afterwards. The center of each drawing should be on the appropriate hour-minute point of the timeline. BE SURE that you have all the Moons oriented the same way, with the "three toed footprint" to the right. Even if the Moon was rising or setting, it was actually moving more or less straight eastward so we are not plotting the Moon as you saw it but as it moved along the ecliptic towards the East. See Figure 2 for a the record made during a deep partial eclipse. The circular edge of the umbra is clearly visible. Now to determine how big that cross section is.

Figure 3 shows the geometry. One needs to find the proper tangent at two or more positions on the umbral edge, three is better and four or more is even more accurate. A proper tangent will have identical angles between the tangent line and curve of the umbral shadow at equal distances from the place where the tangent line touches the umbral edge. Once you have that, draw a perpendicular towards the center of the umbra. Ideally all the perpendiculars would intersect at the same place but because drawings are not perfect and the umbral edge itself is not sharp you should see some spread of intersecting points. Average them out in some way and use a drawing compass to draw a circle that best covers the arc of the umbral edge, then draw the complete shadow circle.

Figure 3. Find tangent and perpendicular lines to locate the center, and find the size, of the umbra of earth's shadow.



Knowing that the Moon drawings represent the Moon's size, one-half degree, you can then determine the diameter of the umbral shadow by direct proportion to that scale. You should get a value that has the umbra being around 2.67 times the size of the Moon, or 1.33 degrees Alternatively, the Moon is around 37% the diameter of the shadow cone of the earth at that distance.

Activity 2 — Size of the Moon and its distance.

Any object has an angular size equal to its diameter in miles divided by its distance from us in miles; the angular size expressed in radians, not degrees. Now, recall that as the distance gets larger, the shadow cone cross section XSD gets smaller. We've measured the size of the umbra, 1.33 degrees in our example. In the box on the next page we take that basic definition of angular size of an object and substitute in Equation 2 on top and Equation 1 on the bottom, and general Equation 3, the f equation at the end

$$\text{Angular Size (A)} = \frac{\text{XSD (cross sectional diameter)}}{\text{Distance to Cross Section}} = \frac{7926 \times (1-f)}{7926 \times 109 \times f} \Rightarrow f = \frac{1}{[(A * 108)+1]} \quad \text{(Eq. 3)}$$

of the chain of mathematics. That is what we need.

Let's take a typical scenario. The shadow cone cross section size we found to be 1.33 degrees. (Use the shadow-to-diameter proportion you found in the first eclipse article in place of 108, and your umbra size from your observations and the circle you made!) To convert any degree value to radians, divide it by 57.3 degrees per radian; this makes the angular size of the shadow .0233 radians. Plugging this value for A into Equation 3 gets us f = 1/ [(0.0233 x 108]+1) or 0.282, 28% of the way from Earth to the end of the cone. Multiply f by the shadow length 856,000 miles makes the Moon about 241,000 miles away. We have found the Moon's distance! Actually, because the Sun still appears to move eastward along the ecliptic, the shadow zone also moves, we underestimate the moon's real distance slightly.

Activity 3 — The Moon's Diameter

This is simpler. Knowing that the Moon (and the umbral cross section) is 28% of the way out, the size of the shadow we see on the Moon is (from Equation 2) 7926 * 1-f, or 7926 x (1-.28), yielding 5706 miles across. As the shadow is 2.67 times the Moon's size, the Moon must be 5706 / 2.67, or 2137 miles across, about 1% smaller than its actual 2160-mile size!

We clearly *can* determine size and distance of the Moon to within a few percent accuracy! How clever those ancient Greek geometers were!

After this December's eclipse, when can you try this again, or use it as an astronomy lab exercise? Here are the dates, types and general locations of the next three partial or total lunar eclipses. Penumbral eclipses are not usable for this project. See also http://eclipse.gsfc.nasa.gov/eclipse.html.

Date	Type	Continents Where Viewable
15 June 2011	Total	S. America, Europe, Asia, Australia
10 Dec. 2011	Total	Europe, Asia, E. Africa, Australia, North America
4 June 2012	Partial (37%)	Asia, Australia, the Americas

Astronomical Teachniques - A Potpourri of Ideas

The Actual Places for the Zodiac Constellations.

Any teacher who sets up a model solar system eventually wants to show how the Sun is in front of a zodiac constellation for some month. However the zodiac constellations not only really number 13, not 12, but also are of unequal sizes. Having representations of the constellations equally spaced around the Sun is just false! So where *do* you place the drawings?

To answer this question I found the borders between the constellations along the ecliptic and then simply found the center of the constellation halfway between them!

Place your constellation card, drawing or name at the following ecliptic longitudes (or pick your 0 degrees at some place around your solar system and measure counterclockwise). Constellation width in degrees is in the parentheses.

Pisces—10, (36)	Libra—230, (23)
Aries—40, (25)	Scorpius—245, (7)
Taurus—71, (37)	Ophiuchus—257, (18)
Gemini—104, (28)	Sagittarius—283, (34)
Cancer—128, (20)	Capricornus—314, (27)
Leo—156, (36)	Aquarius—340, (25)
Virgo—196, (44)	

Have a teach-nique? Send it to editor@toteachthestars.net