Illuminating Science With Darkness 1 - Shadow Cones

- Larry Krumenaker

It only takes a tennis ball to calculate how far the Moon is. Here are two exercises you can use before and after any umbral lunar eclipse, such as the total eclipse in December 2010.

I give you a tennis ball, a marble, and a basketball. Can you tell me which one will have the longest shadow? Of course you can, that's easy. So here's a tougher question. What is the relationship between the length of a shadow and the size of the object? Can you predict how long the shadow would be?

Our Sun causes objects to have shadows but we are neither so close that the Sun's light will come from many different angles, like it might at or within Mercury's orbital distance, nor are we so far, as Pluto, where the light is essentially a pencil beam of parallel rays. Wherever you are, in terms of distance from the Sun, all objects will have the same kind of shadow, and in a certain proportion. This proportion will help us find the distance to the Moon.

Activity 1: Shadow Cones

Because the Sun does send us *nearly* parallel rays, the shadow of the Earth is many times longer than the earth is wide, but exactly how long is that shadow. The fact that the Sun's rays are not parallel, and the Sun actually has some size to it, about one-half degree, means everything here is illuminated from a spread of light beams coming from slightly different angles. That means the space behind the earth, or anything else, has a dark, central lightless region (known as the umbra). Those different angles means the umbra is consequently cone shaped. See Figure 1. Surrounding it is a partially shadowed, partially lit up volume of space, called the penumbra. Here light from some of the Sun's beams gets through but other rays are blocked by the earth (or object).

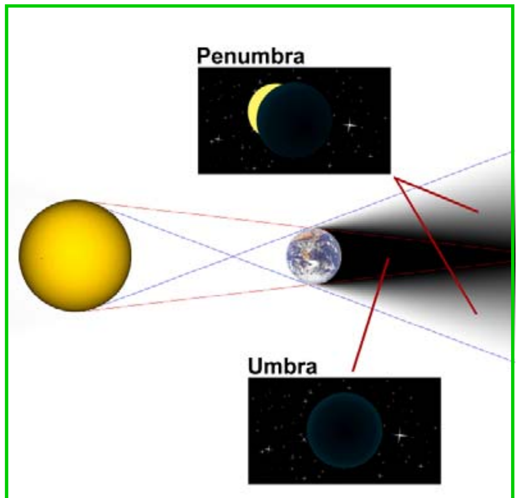

Figure 1. Parts of Earth's shadow.

The natural next questions are how long is the cone, and where does it end and the shadow become *all* penumbra?

To answer these questions we must return to the various spheres mentioned earlier. This experiment works best when the Sun is low, within an hour of rising or setting. That way the cones stretch out horizontally.

Place a ball so that you can cast its shadow on a white piece of paper or screen, at some height above the ground. If close to the ball, you will see a large dark circular zone surrounded by a small, lighter gray area (Figure 2a). The center darkness is the umbra of the ball...but everyone immediately notices that it is fuzzy edged, not sharp. In fact, if you look at the shadow of anything, including yourself, you'll notice the edges aren't sharp either. This is a direct consequence of the fact that the Sun is not a point of light but


Fig. 2a. Ball's shadow and support, mostly umbra.

As the card or screen is moved away from the ball, the umbral zone gets smaller within the shadowed area. At what distance


Fig. 2b. Umbra, surrounded by penumbra, going...

do the umbra and penumbra merge? This is where the end of the shadow cone is located.

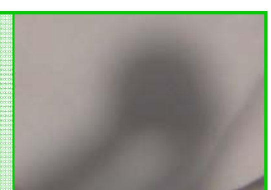
Fig. 2c. ...Gone! Even some sunlight is in the center.

a disk and so light comes from a multitude of directions. Move the ball away until the umbra and penumbra seem to merge. Measure this distance, and do it at least three times to get a good average because there is a fair amount of uncertainty to this measurement! I usually have students work in teams of three, one ball holder, one card holder and one measurer/observer of the umbra. Everybody switches roles so that you get nine good measures if everyone does it three times, or three if everybody does it once.

Using spheres of at least three different diameters confirms the original question, bigger spheres cast longer shadows. But how much does the length depend on the size of the sphere, exactly? To answer this, take the average length and divide by the diameter of sphere. Amazingly enough, no matter what size sphere you have, the shadow length is always around 108 times the size of the sphere! Simply put, anything at the Earth's distance from the Sun casts a shadow that is 108 times longer than the size of the object. And that includes the Earth, and the Moon, as well!

So how big is the earth's shadow? Multiply the earth's diameter (7927 miles or 12,756 kilometers)

by 108! How big is the Moon's shadow? Also, 108 times as long as its diameter. If you know the latter, you'll know the former….OR...recall that the Moon's shadow just barely touches earth during a total solar eclipse. If you can find out the Moon's diameter by some other means, then you can find its distance.

A caveat. The 108 is an average value. It can change by about up to 10% because the earth's distance is not constant because of Earth being in an elliptical orbit. This exercise makes a great prelude to watching a lunar eclipse and using the earth's shadow to find the Moon's size and or distance, as in the last of the exercises here (Page 24). It is best to try and get an accurate ratio of cone length to sphere size in the two weeks prior to the lunar eclipse, and use that value.

There is a second way to determine this ratio and it can be done with the Sun a little higher than when using it to make horizontal shadow cones. You need a sphere, preferably a tennis ball or something smaller and solid and opaque, and a solar filter, preferably the old welder's #14 glass ones—the Mylar solar filters don't do this quite as easily, in my experience.

Set the ball on a thin post, such as on a nail which is then pounded partially into a thin piece of wood, and put it as high in the air as you can manage. I first did this with the ball on the edge of a rain gutter on a second floor roof line. With the solar filter blocking both eyes, line up your eye, the ball, and the Sun. Move yourself towards or away from the ball until the tennis ball just barely blocks out the sunlight. Measure from ball to eye, and divide that distance by the diameter of the tennis ball. You were in the cone shaped shadow of the tennis ball, and the ratio should be also 108.

If you do this method, you MUST assure eye safety. The welder's glass must never fall off or out of the way as one is looking at the Sun (can you also get a welder's helmet to put the filter in?) and the other eye must always be blocked by something at all times as well, perhaps an eye patch.

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Eclipse on December 21, 2010.

The Moon will go through the earth's shadow during this night. On the diagram to the right, you can see the Moon going through the penumbra first, then wholly into the umbra, passing out the other side of the penumbra and then into the clear. The UT times are the times in Great Britain, the standard reference point. For East Coast US, the positions are the hourly spots. The Moon enters the umbra at 6:32 UT (1:32 AM EST), totally immersed at 7:40UT (2:40 AM), begins to exit at 8:53 UT (3:53AM) and wholly in penumbral shadow at 10:01 UT (5:01AM, just before sunrise). Other places, adjust times accordingly.